Section 2-3, Mathematics 108

Information from the Graph of a Function

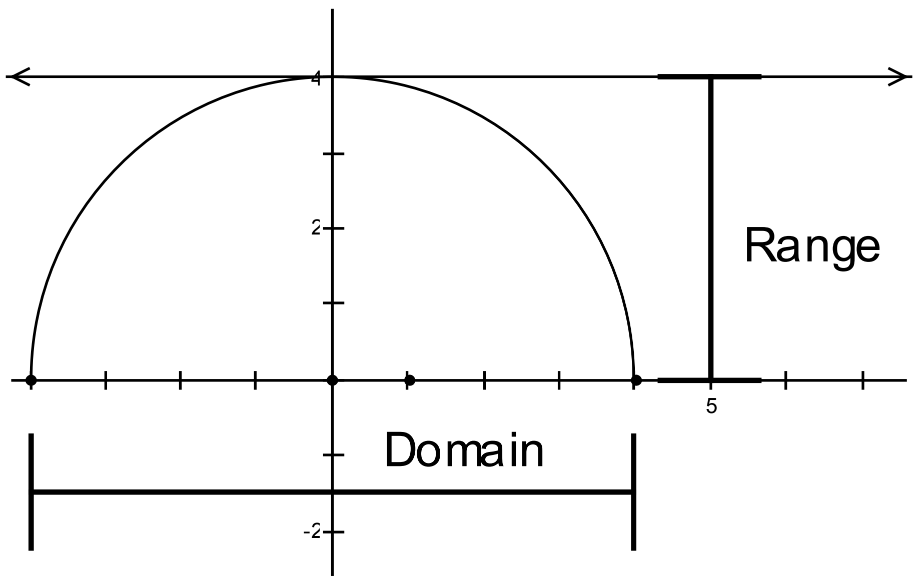
Getting the Domain and Range of a function from a graph.

The domain and range of a function can sometimes be obtained by just looking at a graph.

Example:

This is the graph of a half circle, which is a function.

Note that unlike a full circle it passes the vertical line test.



You see from the graph that the Domain = $\{x : -4 \le x \le 4\}$

and the Range = $\{y : 0 \le y \le 4\}$

Solving Graphically

The book puts some emphasis on solving a problem graphically.

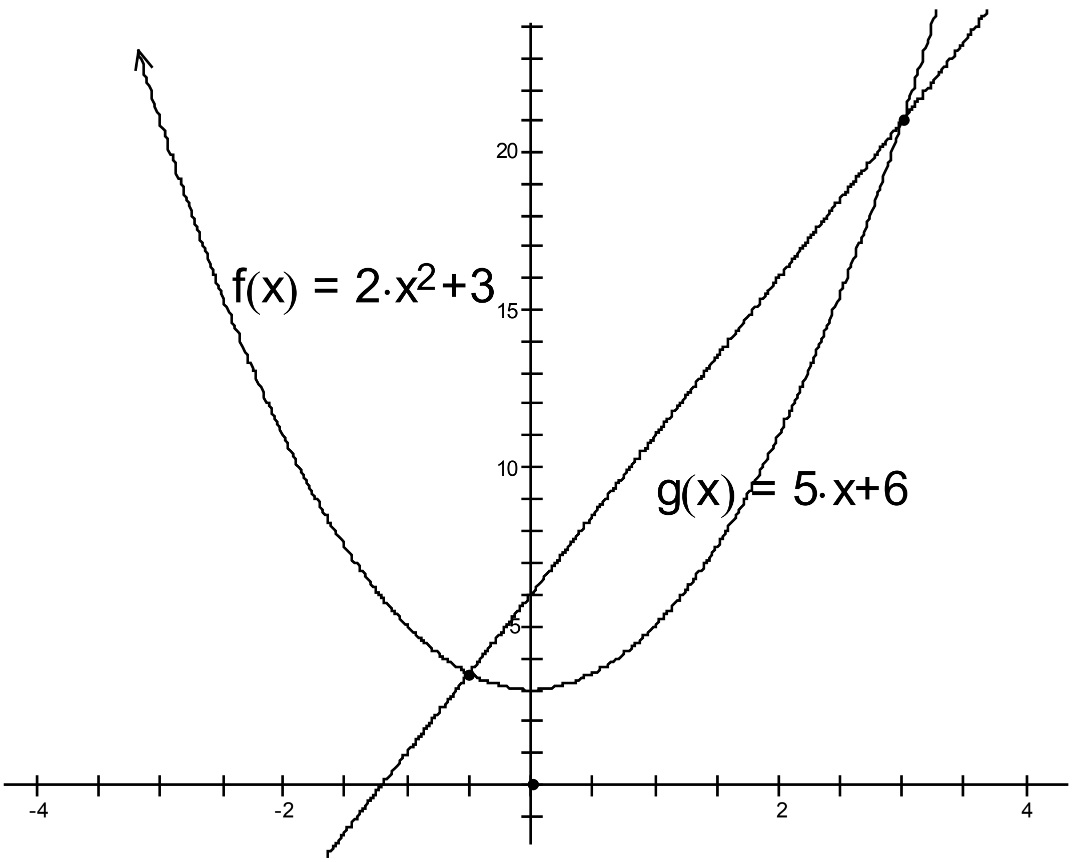
I will disagree somewhat and say that this is generally a poor way to solve a math problem, however sometimes it will add to your understanding of the problem.

Example:

What is the solution to the equation $2x^2 + 3 = 5x + 6$.

You can think of the left side of the equation as the graph of a parabola, and the right side as the graph of a line.

Where the two curves meet will be the solution.



You an see from the graph that the solution will be two points.

However solving this using a graph is a poor way to go about it.

It would be much better to solve the equation algebraically, or using a computer to search for the solution.

Increasing functions and Decreasing functions

An increase function is a function increases from left to right.

We can write this algebraically as

For all $a,b \in Domain(f)$, $a < b$ implies that $f(a) < f(b)$.

Similarly a decreasing function decreases from left to right:

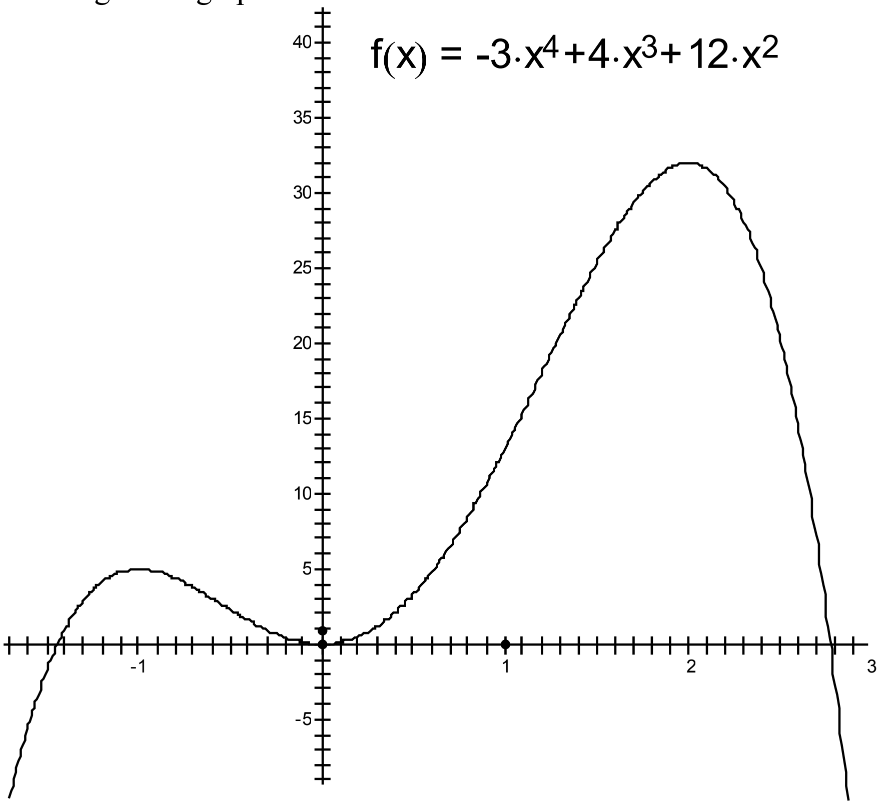
For all $a,b \in Domain(f)$, $a > b$ implies that $f(a) > f(b)$.

Sometimes increasing and decreasing functions are called **monotonic.**

Increasing and Decreasing on an Interval

Similar to the definitions of increasing and decreasing functions, we define a function to be increasing or decreasing on an interval if $f(a) < f(b)$ or $f(a) > f(b)$ on that interval.

Example:
Let $f(x) = -3x^4 + 4x^3 + 12x^2$
Looking at the graph we see



From the graph it looks like the function increases below -1 to about -1.
Then it decreases from about -1 to zero.
Then it increases again to about 2 and then decreases thereafter.
So we would say

$(-\infty, -1]$	Increasing
$[-1, 0]$	Decreasing
$[0, 2]$	Increasing
$[2, \infty)$	Decreasing

We do not have the tools yet to determine these intervals algebraically, but you will learn how in Calculus.

Maximums and Minimums

In the previous example, you might have noticed that except for two ends of the graph, the increasing and decreasing intervals begin and end at the top and bottoms of bumps on the graph, which are called local **Minimums** and **Maximums**.

We can define a local maximum of *f* on an interval I at *a* as

$$Max = f\left(a\right) | \, a \in I, \forall x \in I, f\left(a\right) \geq f\left(x\right)$$

Similarly we can define a local minimum of *f* on an interval I at *a* as

$$Min = f\left(a\right) | \, a \in I, \forall x \in I, f\left(a\right) \leq f\left(x\right)$$

If you have a graphing calculator, it may have a way to calculate minimums and maximums.